MANGALORE UNIVERSITY
Department of Mathematics

Admission to M. Sc. Mathematics - Entrance Test - 2023

Do not open this booklet until you are asked to do so

Duration : 90 Minutes **Maximum Marks : 50**

Name of Candidate (in block letters)	:	
Application Number		
Signature of the Candidate		Signature of the Invigilator

General Instructions

1. This Question Paper contains 25 multiple choice objective type questions.
2. **All questions carry equal marks. Two marks for each correct answer**.
3. Each question in this booklet is followed by four probable answers which are indicated by letters (A), (B), (C) and (D). Mark the letter indicating your correct answer on the box provided (near the question) using pen. More than one letter in the box will be treated as wrong answer.
4. All rough work should be done in the space provided at the end of the booklet.
5. Calculator, mobile phone, pager, log table etc. are not allowed.
6. Malpractices, if any will disqualify your candidature.
7. All candidates must return this booklet to the invigilator at the end of the test.
8. Invigilator will not entertain any sort of questions after the commencement of the test.

For office use only

Number of Correct answers		**Total marks out of 50**	

Signature of the Valuer

Do not write on this page

1. How many symmetric relations can be defined on a set with 3 elements?

(A) 6. (B) 16. (C) 32. (D) 64.

2. Which of the following function $f : \mathbb{R} \to \mathbb{R}$ is injective ?

(A) $f(x) = (x-2)^2$. (B) $f(x) = \sin x$. (C) $f(x) = -x^3$. (D) $f(x) = |x+1|$.

3. Which of the following set of real matrices is not a group?

(A) All $n \times n$ diagonal matrices under matrix addition.

(B) All $n \times n$ upper-triangular matrices under matrix multiplication.

(C) All $n \times n$ upper-triangular matrices under matrix addition.

(D) All $n \times n$ matrices with determinant either 1 or -1 under matrix multiplication.

4. For the group homomorphisms $\phi : \mathbb{Z} \to \mathbb{Z}_5$ defined by $\phi(1) = 4$, the kernel is

(A) $5\mathbb{Z}$. (B) $\mathbb{Z}$. (C) $\mathbb{Z}_5$. (D) $\{0\}$.

5. The digit in the unit place of decimal expansion of 3^{2023} is

(A) 1. (B) 3. (C) 7. (D) 9.

6. Let A be an $n \times n$ matrix with integer entries. Then $\det(-A) =$

(A) $-\det A$. (B) $(-1)^n \det A$. (C) $\det A$. (D) $(-1)^n$.

7. The distance from the point $(1, 2, 3)$ to the $xy-$plane is

(A) 1. (B) 2. (C) 3. (D) $\sqrt{5}$.

8. Let C be the line segment joining $(0,0,0)$ to $(1,1,1)$. Then $\displaystyle\int_C (x-3y^2+z)dx =$

(A) 0. (B) 1. (C) $\sqrt{3}$. (D) 3.

9. Let $f : X \to Y$ be any function and A, B be subsets of X, Y, respectively. Then

(A) $f^{-1}(f(A)) \subseteq A$. (C) $f(f^{-1}(B)) \supseteq B$.

(B) $f^{-1}(f(A)) \supseteq A$. (D) $f\,(X - A) = Y - f(A)$.

10. Let $f(x) = min\{\sin x, \cos x\}$ for all $x \in \mathbb{R}$. Then

(A) f is differentiable on $\mathbb{R}$.

(B) f is differentiable except at finitely many points.

(C) f is not continuous at any point of $\mathbb{R}$.

(D) f is continuous on $\mathbb{R}$.

11. $\int_{-1}^{1} |2x+1|dx =$

(A) $\frac{5}{2}$. (B) $\frac{1}{2\sqrt{2}}$. (C) $\frac{1}{4}$. (D) $\frac{1}{4\sqrt{2}}$.

12. $\displaystyle\lim_{n\to\infty} \left(\frac{n+1}{n\sqrt{n}}\right) =$

(A) 0. (B) 1. (C) -1. (D) ∞.

13. Harmonic conjugate of the function $u(x,y) = y^3 - 3x^2y$ is

(A) $x^3 + 3x^2y$. (B) $x^3 + 3xy^2$. (C) $x^3 - 3xy^2$. (D) $x^3 - 3x^2y$.

14. Which of the following is a linear differential equation?

(A) $y' + y^2 = 0$.

(B) $y'' + 2x^2 y = x^3$.

(C) $y'' + \cos y = 0$.

(D) $y' + \sin y = 4$.

15. The orthogonal trajectories of the family of curves $x^2 + y^2 = c^2$ is

(A) family of straight lines passing through origin.

(B) a family of circles passing through origin.

(C) the family of circle $x^2 + y^2 = c^2$ itself.

(D) a family of ellipses passing through origin.

16. Let $\{a_n\}$, $\{b_n\}$ and $\{c_n\}$ be sequences of real numbers such that $b_n = a_{2n}$ and $c_n = a_{2n+1}$. Then the convergence of $\{a_n\}$ implies

(A) $\{b_n\}$ is convergent, but $\{c_n\}$ need not converge.

(B) $\{c_n\}$ is convergent, but $\{b_n\}$ need not converge.

(C) both $\{b_n\}$ and $\{c_n\}$ are convergent.

(D) nothing about the convergence of $\{b_n\}$ or $\{c_n\}$.

17. Let f, $g : \mathbb{R} \to \mathbb{R}$ be functions satisfying the conditions $|f(x) - f(y)| \geq |x - y|$ for all $x, y \in \mathbb{R}$ and $|g(x) - g(y)| \leq |x - y|$ for all $x, y \in \mathbb{R}$. Then

(A) f is one-to-one and g is continuous.

(B) f is continuous and g is one-to-one.

(C) both f and g are one-to-one.

(D) both f and g are continuous.

18. Consider the statements (i) and (ii) below:

(i) $f,\, g : \mathbb{R} \to \mathbb{R}$ be functions satisfying the condition that $f(n) = g(n)$ for all $n \in \mathbb{Z}$. Then $f = g$

(ii) $\int_1^\infty \log x \, dx$ is finite.

Then

(A) (i) is True and (ii) is False.

(B) (i) is False and (ii) is False.

(C) (i) is False and (ii) is True.

(D) (i) is True and (ii) is True.

19. The sum of the series $\dfrac{1}{1\cdot 2} + \dfrac{1}{2\cdot 3} + \dfrac{1}{3\cdot 4} + \cdots + \dfrac{1}{100\cdot 101}$ is

(A) $\dfrac{99}{101}$.

(B) $\dfrac{98}{101}$.

(C) $\dfrac{99}{100}$.

(D) None of the above.

20. Which of the following is a subspace of the vector space $\mathbb{R}^2$ over $\mathbb{R}$?

(A) $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$.

(B) $\{(x, y) \in \mathbb{R}^2 : x + y = 0\}$.

(C) $\{(x, y) \in \mathbb{R}^2 : x + y = 1\}$.

(D) $\{(x, y) \in \mathbb{R}^2 : y = x^2\}$.

21. If $A = (a_{ij})$ is the 6×6 matrix with $a_{ij} = 1$ for all $1 \le i, j \le 6$, then

(A) $\det A = 1$, Rank $A = 1$.

(B) $\det A = 0$, Rank $A = 1$.

(C) $\det A = 6$, Rank $A = 5$.

(D) $\det A = 1$, Rank $A = 5$.

22. What is the inverse of 'a' in the group $(\mathbb{Z},\ *)$, where $a * b = a + b + 1$ for all $a, b \in \mathbb{Z}$?

(A) $a - 2$.

(B) $-a - 2$.

(C) 0.

(D) -2.

23. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation such that $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$, then $T\left(\begin{bmatrix} -1 \\ -2 \end{bmatrix}\right) =$ $\square$

(A) $\begin{bmatrix} -5 \\ -2 \\ 1 \end{bmatrix}$. (B) $\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$. (C) $\begin{bmatrix} -5 \\ -2 \\ 0 \end{bmatrix}$. (D) $\begin{bmatrix} -5 \\ -3 \\ 1 \end{bmatrix}$.

24. The number of elements in the set $\{\{a, b\}, \{c\}\}$ is $\square$

(A) 1. (B) 2. (C) 3. (D) 4.

25. The Laplace transform of $e^{-2t} \sin t$ is $\square$

(a) $\dfrac{1}{s^2 + 4s + 5}$. (b) $\dfrac{s + 2}{s^2 + 4s + 5}$. (c) $\dfrac{1}{s^2 - 4s + 5}$. (d) $\dfrac{s - 2}{s^2 - 4s + 5}$.

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